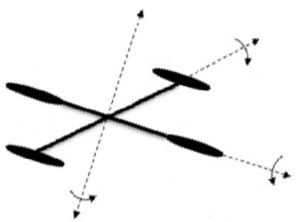
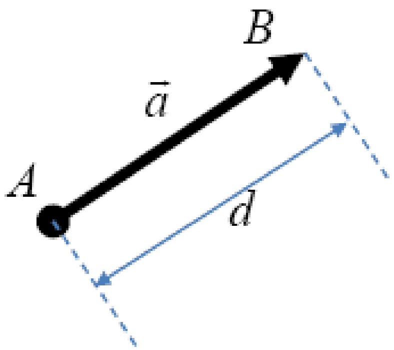
7장 Vectors

7.1 7.2 벡터의 기초



벡터의 정의

스칼라(scalar) 크기만 가지고 있는 어떤 물리량 (길이, 온도, 전압 등)
벡터(vector) 크기와 방향을 가지고 있는 물리량 (속도, 힘 등)



A 시점 : 벡터가 시작되는 지점
B 종점 : 벡터가 끝나는 지점
d 벡터의 크기

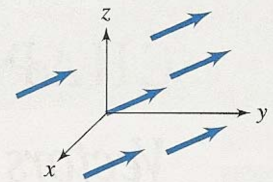
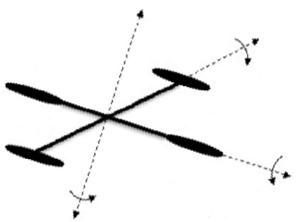
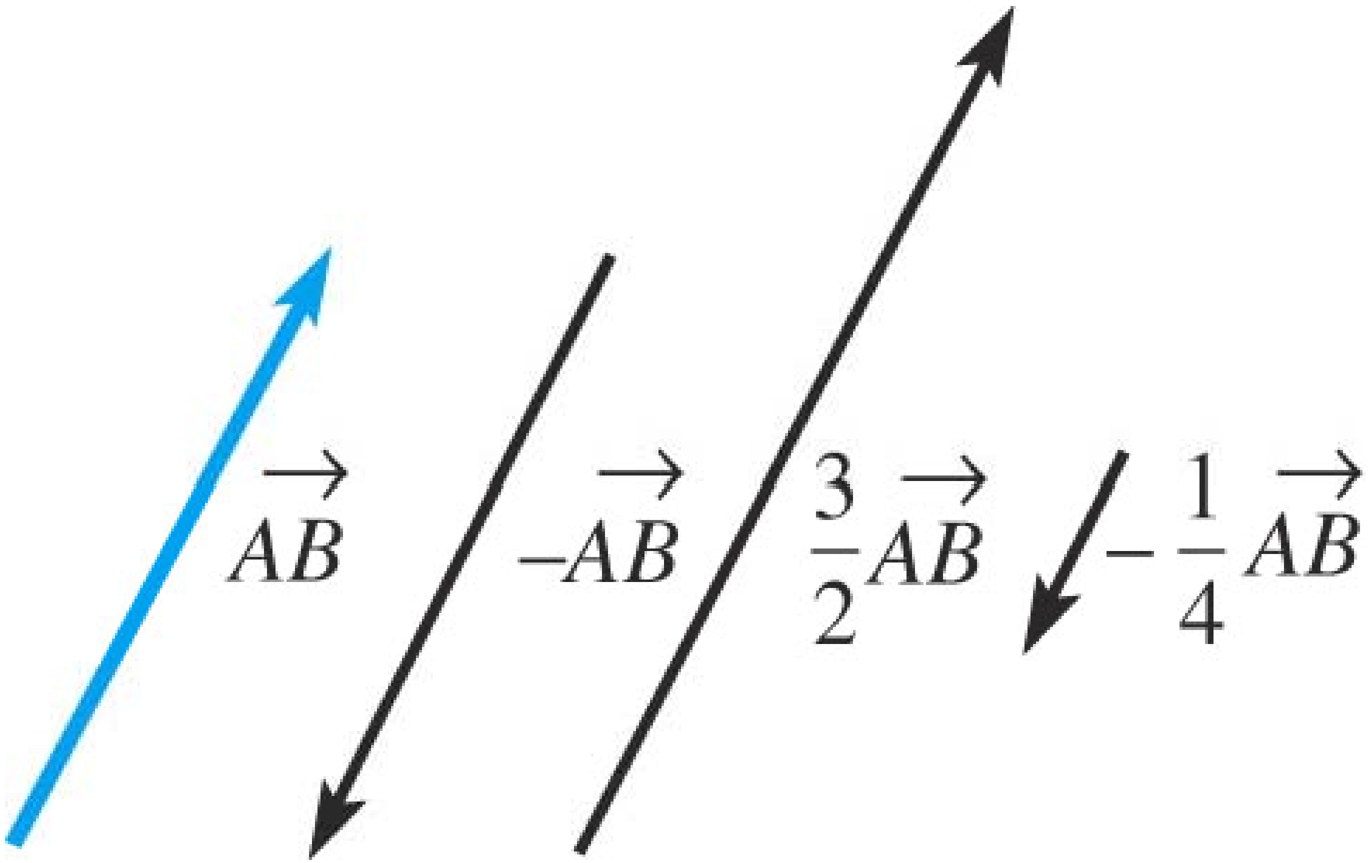


FIGURE 5.3 *Parallel representations of the same vector.*

벡터의 등가 크기와 방향이 같으면 두 벡터는 같다. 즉 벡터는 시점과 종점이 달라도 평행이동으로 겹쳐질 수 있다면, 같은 벡터이다.




$\overrightarrow{AB}$
$-\overrightarrow{AB}$
$\frac{3}{2}\overrightarrow{AB}$
$-\frac{1}{4}\overrightarrow{AB}$

예제 4 두 점 사이의 벡터

점 $P_1(4, 6, -2)$와 $P_2(1, 8, 3)$ 사이의 벡터 $\overrightarrow{P_1P_2}$를 구하라.

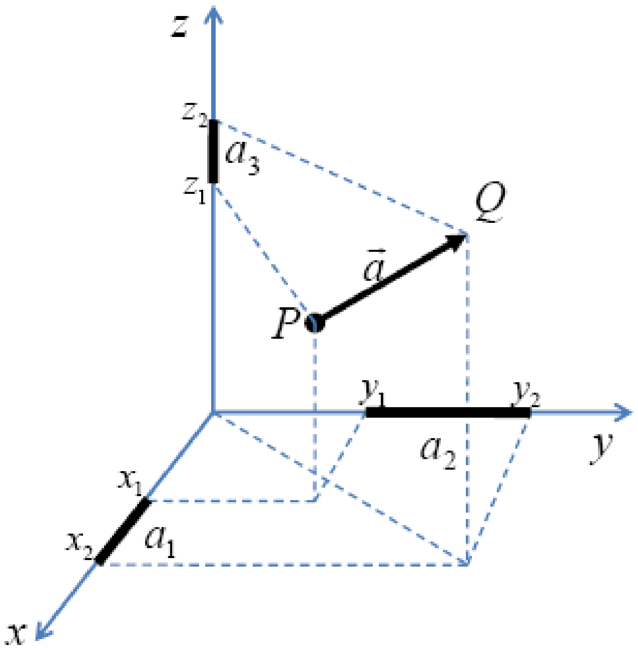
풀이 점 P_1과 P_2의 위치벡터가 $\overrightarrow{OP_1} = <4, 6, -2>$와 $\overrightarrow{OP_2} = <1, 8, 3>$이라 하면, (3)으로부터

$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \langle 1-4, 8-6, 3-(-2) \rangle = \langle -3, 2, 5 \rangle$$

이다. ❑

- 벡터의 성분

점 $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$ 을 각각 시점과 종점이라고 하면,



벡터의 성분 $a_1, \ a_2, \ a_3$

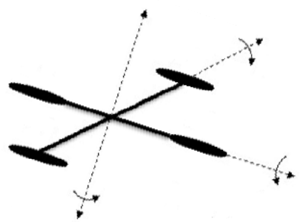
$$a_1 = x_2 - x_1$$

$$a_2 = y_2 - y_1$$

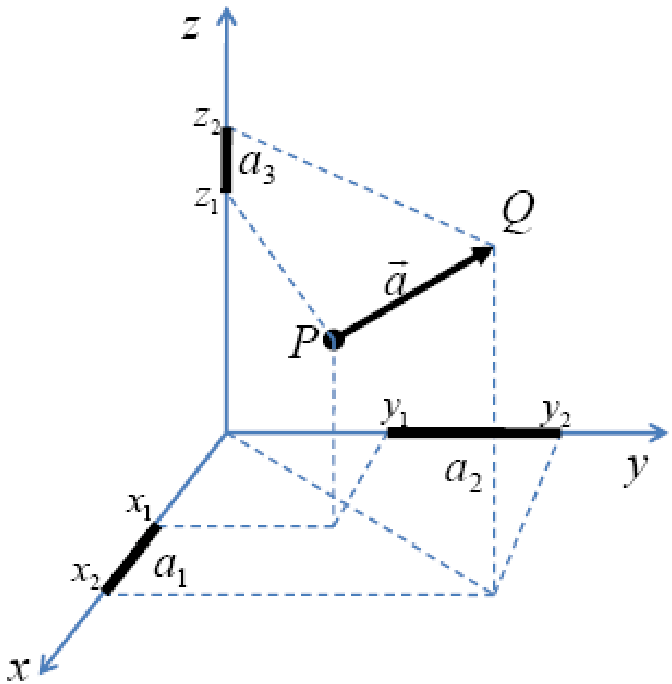
$$a_3 = z_2 - z_1$$

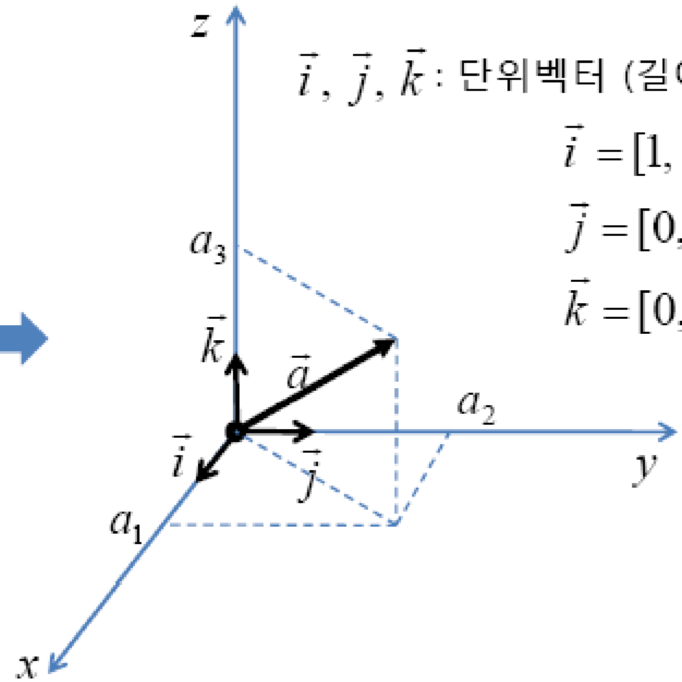
벡터의 성분 표현법 $\vec{a} = [a_1, \ a_2, \ a_3]$

벡터의 크기 $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$



- 위치 벡터 (시점을 원점으로 평행이동시킨 것)





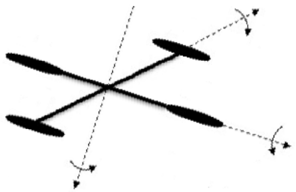
$\vec{i}, \vec{j}, \vec{k}$: 단위벡터 (길이가 '1')

$\vec{i} = [1,\ 0,\ 0]$

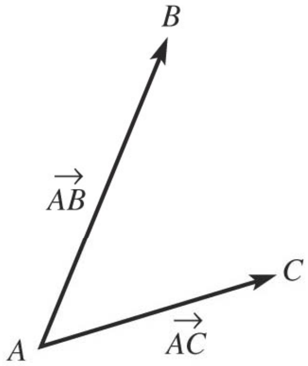
$\vec{j} = [0,\ 1,\ 0]$

$\vec{k} = [0,\ 0,\ 1]$

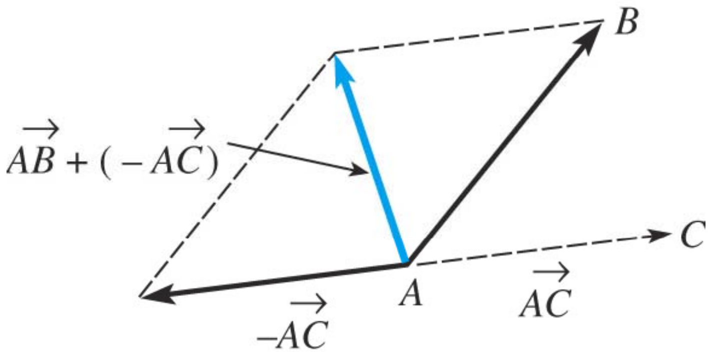
$$\vec{a} = [a_1,\ a_2,\ a_3] = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$



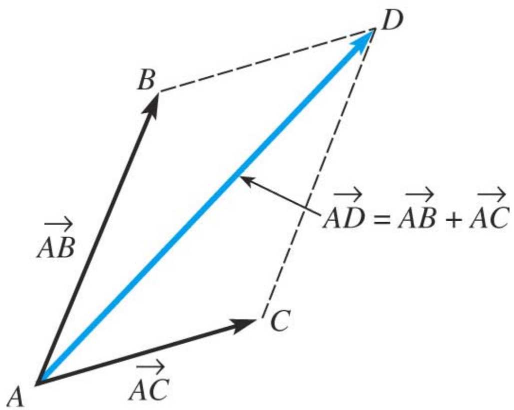
- 벡터의 덧셈과 뺄셈



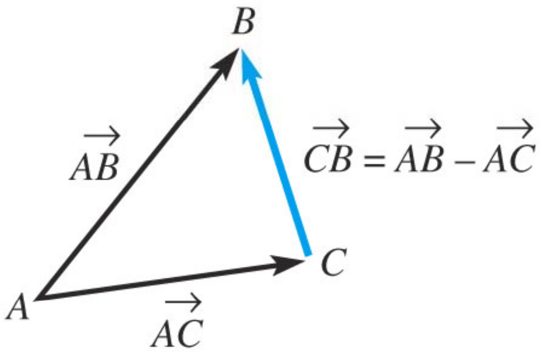
(a)



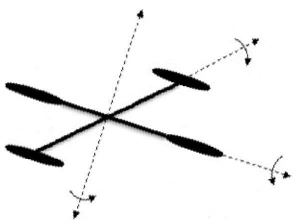
(a)



(b)



(b)



DEFINITION

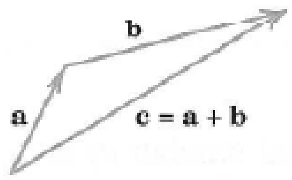


Fig. 168. Vector addition

Addition of Vectors

The **sum a + b** of two vectors $\mathbf{a} = [a_1, a_2, a_3]$ and $\mathbf{b} = [b_1, b_2, b_3]$ is obtained by adding the corresponding components,

$$(3) \qquad \mathbf{a} + \mathbf{b} = [a_1 + b_1, \quad a_2 + b_2, \quad a_3 + b_3].$$

Geometrically, place the vectors as in Fig. 168 (the initial point of **b** at the terminal point of **a**); then **a + b** is the vector drawn from the initial point of **a** to the terminal point of **b**.

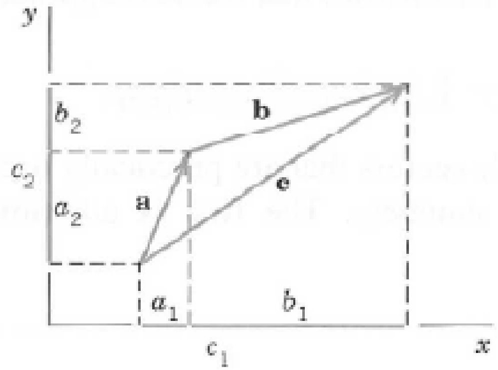


Fig. 170. Vector addition

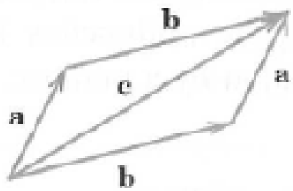


Fig. 171. Cummutativity of vector addition

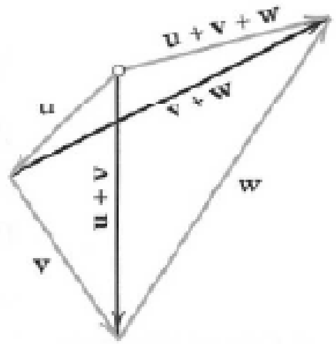
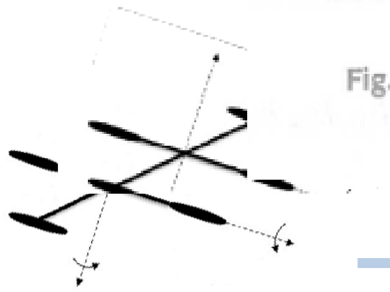


Fig. 172. Associativity of vector addition



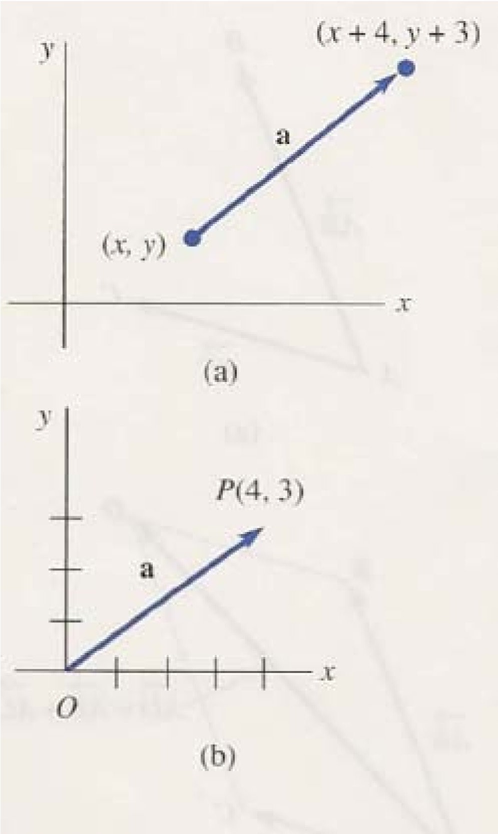
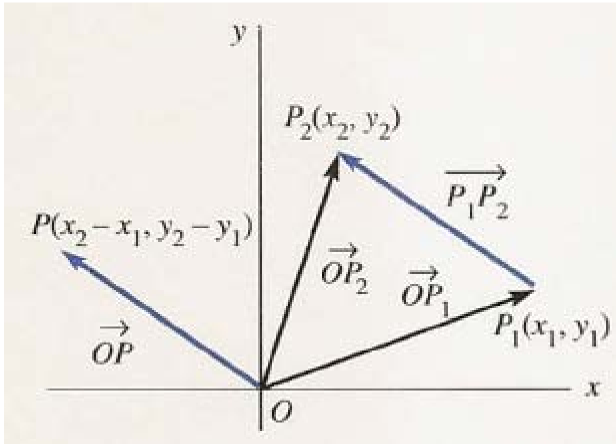
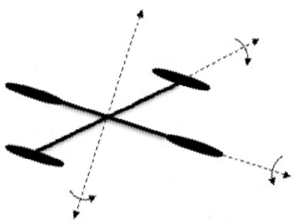


그림 7.7 (a)와 (b)의 벡터는 같다

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \langle x_2 - x_1, y_2 - y_1 \rangle$$





The algebraic sum of two vectors is defined as follows.

DEFINITION 5.4 ***Vector Sum***

The sum of $\mathbf{F} = (a_1, b_1, c_1)$ and $\mathbf{G} = (a_2, b_2, c_2)$ is the vector

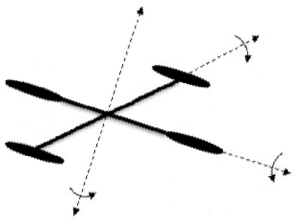
$$\mathbf{F} + \mathbf{G} = (a_1 + a_2, b_1 + b_2, c_1 + c_2).$$

That is, we add vectors by adding respective components. For example,

$$(-4, \pi, 2) + (16, 1, -5) = (12, \pi + 1, -3).$$

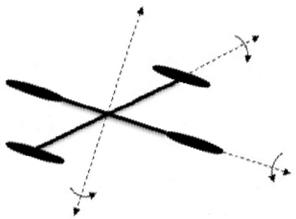
If $\mathbf{F} = (a_1, b_1, c_1)$ and $\mathbf{G} = (a_2, b_2, c_2)$, then the sum of $\mathbf{F}$ with $-\mathbf{G}$ is $(a_1 - a_2, b_1 - b_2, c_1 - c_2)$. It is natural to denote this vector as $\mathbf{F} - \mathbf{G}$ and refer to it as "$\mathbf{F}$ minus $\mathbf{G}$." For example, $(-4, \pi, 2)$ minus $(16, 1, -5)$ is

$$(-4, \pi, 2) - (16, 1, -5) = (-20, \pi - 1, 7).$$



Let $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ be vectors and let α and β be scalars. Then

1. $\mathbf{F} + \mathbf{G} = \mathbf{G} + \mathbf{F}$.
2. $(\mathbf{F} + \mathbf{G}) + \mathbf{H} = \mathbf{F} + (\mathbf{G} + \mathbf{H})$.
3. $\mathbf{F} + \mathbf{O} = \mathbf{F}$.
4. $\alpha(\mathbf{F} + \mathbf{G}) = \alpha\mathbf{F} + \alpha\mathbf{G}$.
5. $(\alpha\beta)\mathbf{F} = \alpha(\beta\mathbf{F})$.
6. $(\alpha + \beta)\mathbf{F} = \alpha\mathbf{F} + \beta\mathbf{F}$. ■

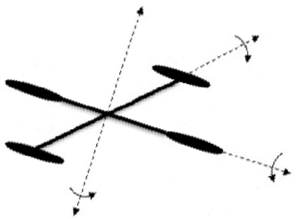


예제 2 두 벡터의 덧셈과 뺄셈

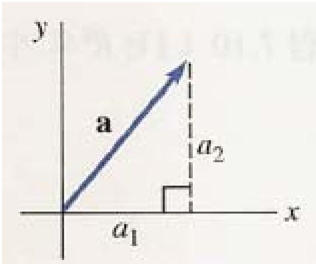
$\mathbf{a} = <1, 4>$, $\mathbf{b} = <-6, 3>$일 때, $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $2\mathbf{a}+3\mathbf{b}$를 구하라.

풀이 (1), (2), (4)에 의하여 다음을 얻는다.

$$\mathbf{a}+\mathbf{b} = \langle 1+(-6), 4+3 \rangle = \langle -5, 7 \rangle$$
$$\mathbf{a}-\mathbf{b} = \langle 1-(-6), 4-3 \rangle = \langle 7, 1 \rangle$$
$$2\mathbf{a}+3\mathbf{b} = \langle 2, 8 \rangle + \langle -18, 9 \rangle = \langle -16, 17 \rangle$$



- 벡터의 크기



$\mathbf{a} = \langle a_1, a_2 \rangle$ 일 때 $\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2}$

예제 5 벡터의 크기

정의 7.2 (*vii*)로부터, **a**=<−2/7, 3/7, 6/7>의 크기는

$$\|\mathbf{a}\| = \sqrt{\left(-\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4+9+36}{49}} = 1$$

- 단위 벡터

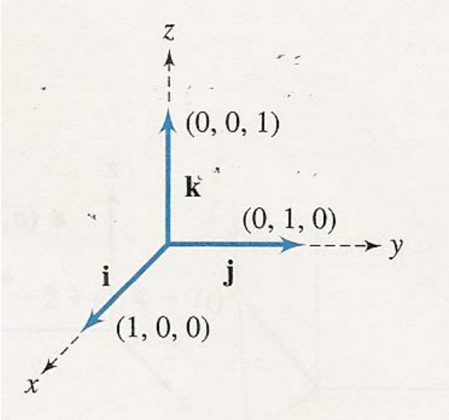


FIGURE 5.7 *Unit vectors along the axes.*

$$\mathbf{i} = (1, 0, 0), \quad \mathbf{j} = (0, 1, 0), \quad \mathbf{k} = (0, 0, 1).$$

$$\mathbf{F} = (a, b, c) = a(1, 0, 0) + b(0, 1, 0) + c(0, 0, 1) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}.$$

$$(-3, 0, 1) = -3\mathbf{i} + \mathbf{k}.$$

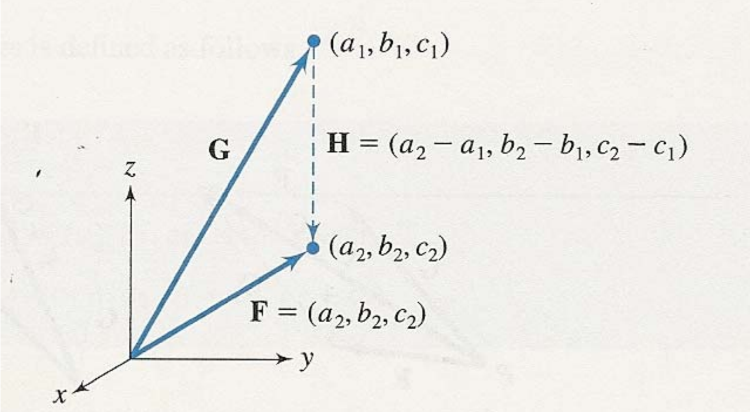


FIGURE 5.9 *The arrow from* (a_1, b_1, c_1) *to* (a_2, b_2, c_2) *is* $(a_2 - a_1)\mathbf{i} + (b_2 - b_1)\mathbf{j} + (c_2 - c_1)\mathbf{k}$.

$$\mathbf{G} + \mathbf{H} = \mathbf{F}.$$

$$\mathbf{H} = \mathbf{F} - \mathbf{G} = (a_2 - a_1)\mathbf{i} + (b_2 - b_1)\mathbf{j} + (c_2 - c_1)\mathbf{k}.$$

예제 3 단위벡터

$\mathbf{a}=<2, -1>$일 때, $\mathbf{a}$와 같은 방향과 반대 방향의 단위벡터를 구하라.

풀이 벡터 $\mathbf{a}$의 크기는 $\|\mathbf{a}\|=\sqrt{4+(-1)^2}=\sqrt{5}$이다. $\mathbf{a}$와 같은 방향의 단위벡터는

$$\mathbf{u}=\frac{1}{\sqrt{5}}\mathbf{a}=\frac{1}{\sqrt{5}}\langle 2, -1\rangle\left\langle\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right\rangle$$

이고, $\mathbf{a}$와 반대 방향의 단위벡터는 다음과 같다.

$$-\mathbf{u}=\left\langle-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle$$

❑

예제 8 일차결합

$\mathbf{a}=3\mathbf{i}-4\mathbf{j}+8\mathbf{k}$, $\mathbf{b}=\mathbf{i}-4\mathbf{k}$ 일 때, $5\mathbf{a}-2\mathbf{b}$ 를 구하라.

풀이 $\mathbf{b}$ 를 3차원 벡터로 간주하여 $\mathbf{b}=\mathbf{i}+0\mathbf{j}-4\mathbf{k}$ 로 쓰자.

$$5\mathbf{a} = 15\mathbf{i} - 20\mathbf{j} + 40\mathbf{k} \quad \text{그리고} \quad 2\mathbf{b} = 2\mathbf{i} + 0\mathbf{j} - 8\mathbf{k}$$

로부터

$$\begin{aligned} 5\mathbf{a} - 2\mathbf{b} &= (15\mathbf{i} - 20\mathbf{j} + 40\mathbf{k}) - (2\mathbf{i} + 0\mathbf{j} - 8\mathbf{k}) \\ &= 13\mathbf{i} - 20\mathbf{j} + 48\mathbf{k} \end{aligned}$$

이다. ❑